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Size and loading mode effects in fracture toughness testing of polymers

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Experimental work is described which examines the applicability of plane-strain fracture toughness testing techniques to several polymers. The fracture behaviour of five polymers was studied using pre-notched test specimens and was characterized by the linear elastic fracture parameter, K_c. Two test geometries and loading modes were used; SEN tension and three-point bending have been investigated by varying both thickness and width. It has been established that the ASTM criterion for plane-strain conditions of $B > 2.5 (K_{c1}/\sigma_y)^2$ is sufficient for SEN bending but not for tension where an extrapolation method is needed. For width effects the BCS model was shown to describe the observations and this with a limiting nominal stress analysis gave quite close agreement with the ASTM criterion $W > 5(K_{c1}/\sigma_y)^2$. This value was shown to be a good estimate of the practical minimum width. There is some evidence that PP, which gives substantial crazing and whitening, can give satisfactory values at sizes about half these limiting values.

Nomenclature

a	crack length.
B	specimen thickness.
B_{min}	minimum specimen thickness.
G_c	strain energy release rate.
$\dot{e}$	strain rate.
K	stress intensity factor.
K_c	critical stress intensity factor.
K_{c1}	plane-strain fracture toughness.
K_{c2}	plane-stress fracture toughness.
M	bending moment.
P	fracture load.
r_p	plastic zone size.
r_{p2}	plastic zone size in plane-stress.
S	span.
T	temperature.
W	specimen width.
W_{min}	minimum specimen width.
$\dot{x}$	cross-head speed.
y	finite width correction factor.
σ	stress.
σ_c	fracture stress.
σ_N	net section stress.
σ_{pc}	gross stress at plastic collapse.
σ_y	yield stress.

1. Introduction

The use of fracture mechanics parameters such as K_c or G_c to characterize the toughness of polymers is now quite widespread [1–3]. For low strength materials this presents few problems since almost any size of specimen and loading mode will give a brittle fracture from which a K_c or G_c can be found. For the tougher materials, where practical interest inevitably concentrates, the extent of plastic deformation is always larger. The comparison of this, as characterized by the plastic zone size, and the specimen dimensions gives rise to the size effects discussed here. The material toughness is best characterized by its value under plane-strain conditions, K_{c1}, and to achieve this stress state the plastic zone must be appreciably less than the specimen thickness. This has been embodied in the ASTM empirical criteria for bend tests [4];

$$B > 2.5(K_{c1}/\sigma_y)^2, \qquad (1)$$

and this will be explored here for both bending and tension tests on five widely different polymers.

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The formulae for calculating K_{c1} assume that the entire deformation process is controlled by linear elastic deformations and to achieve this the other specimen dimensions, crack length a, depth W, and length l, must be such as to assure this linear behaviour and again this can be expressed as a relationship to the plastic zone size and is thus determined by $(K_{c1}/\sigma_y)^2$ so that a width or depth W criterion must be established. In addition this behaviour of the crack is strongly influenced by the sharpness of the original notch formed in the specimen. For brittle materials it is often possible to propagate a stable crack at the tip of the notch by some sort of razor blade method but for the tougher materials such cracks do not form easily. To overcome this notches are often machined in with sharp fly cutters and it is known that some materials require more care than others in this process. Here the specimens for PMMA were razor notched while for the other materials a sharp fly cutter was used to give tip radii of about 13 μm which was found to give consistent results. The effect of varying the notch tip sharpness will be reported elsewhere [5].

The work described here is a continuation of a detailed study on three polyethylenes [6] which showed that the ASTM thickness criterion was adequate and that the stress-limiting width criterion worked well in bending. However, tests on polypropylene have shown behaviour rather at variance with these ideas, particularly in tension [7]. The different forms of yielding process in polymers also cast doubt on the use of the ASTM criterion which is essentially for a constant volume yielding process. In many polymers we have crazing and also diffuse microvoiding both of which give marked increases in volume during deformation and might be expected to give modifications to the criteria. With these factors in mind the following set of materials were chosen for investigation.

1. Polymethylmethacrylate (PMMA); ICI high molecular weight cast sheet (thickness 6 and 10 mm). Chosen as a good example of a polymer giving brittle fractures under most conditions and being easy to notch.

2. Polyvinyl chloride (PVC); unplasticized ICI Darvic 110, calendared sheet (10 and 12.5 mm). A commercially important type of material with reasonable toughness but exhibiting brittle behaviour under many conditions.

3. Polyacetal (PA); Dupont Delrin extruded sheet (11 and 20 mm). A tough "engineering plastic" for which plane-strain toughness values are of practical concern.

4. Nylon, ICI extruded sheet of modified Nylon 66 (11 mm). Again a tough material much used in engineering applications.

5. Polypropylene copolymer (PP); ICI extruded sheet of ethylene–propylene copolymer (20 mm); a toughened material with a distinct second phase [8] which shows considerable microvoiding.

2. Fracture mechanics concepts

Fracture mechanics seeks to establish suitable parameters to characterize the fracture of a cracked body under loading. In linear elastic fracture mechanics (LEFM), the stress field ahead of a sharp crack in a structural member can be characterized by a single parameter, K, the stress intensity factor. The parameter K is sufficient to relate the intensity of the elastic stress–strain field near the crack tip to the loading and the geometry of finite dimensions under different loading conditions using appropriate expressions. These are usually written as [4];

$$K = y\sigma a^{\frac{1}{2}} \tag{2}$$

where y is the geometrical correction factor, σ the gross applied stress and a the crack length.

It has been shown that crack initiation in polymers can be described in terms of the geometry independent parameter, K_c, the critical stress intensity, provided that plastic zone size at the crack tip is small compared to the relevant specimen dimensions (e.g. thickness and width) and the crack length. The size of the plastic zone influences the state of stress so that when the plastic zone is large compared to the specimen thickness, yielding can take place in the thickness direction giving plane stress and a high observed toughness. When the plastic zone size is small, yielding in the thickness direction is constrained by the surrounding elastic material resulting in a plane-strain stress state and a consequent lower-bound fracture toughness K_{c1} (the plane-strain fracture toughness). The main advantage of measuring K_{c1} is that the value may be used to compare the true basic toughness of the materials. Methods of evaluating K_{c1} have been the subject of many recent investigations by the ASTM Special Committee E24 which carried out a comprehensive study of the subject and published a series of reports advocating suitable test procedures. Employing single-edge notch bend and

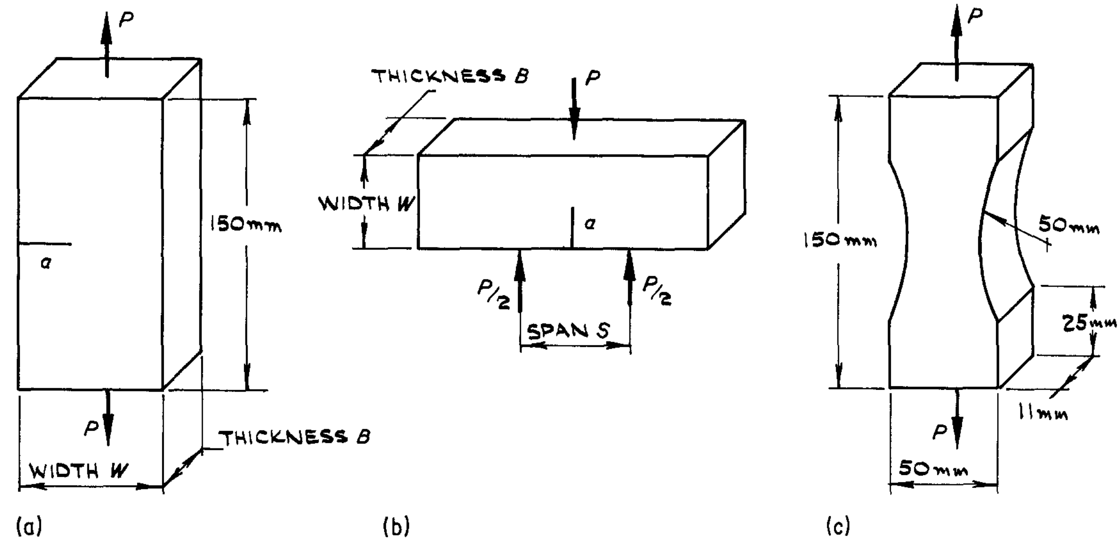


Figure 1 Specimen configuration. (a) SEN tension; (b) SEN three-point bending; (c) yield stress specimen.

compact tension specimens, and taking the plastic zone size in plane-stress condition as $r_p = 1/2\pi\, (K_{c1}/\sigma_y)^2$ where σ_y is the material yield strength, the Committee recommended the following specimen size requirements to ensure elastic plane-strain behaviour [4];

$$a = \text{crack length} > 2.5(K_{c1}/\sigma_y)^2$$

$$B = \text{specimen thickness} > 2.5(K_{c1}/\sigma_y)^2$$

$$W = \text{specimen width} > 5.0(K_{c1}/\sigma_y)^2.$$

The above conditions suggest that for specimen meeting ASTM requirements, the specimen thickness must exceed $15r_p$.

3. Experimental details

3.1. Fracture tests

Single-edge notched (SEN) specimens were prepared for testing in two modes of loading; tension and three-point bending (Figs. 1a and b). The fracture tests were performed on an Instron testing machine and a three-point bend rig consisting of fixed roller supports which could be adjusted to different spans. The tension specimens were clamped (pin loaded) using universal joints which provided the necessary displacement as the crack tip propagated through the specimen width. Tests below room temperature (20° C) were performed in an insulated box cooled by vaporizing liquid nitrogen. The temperature inside the box was monitored by a thermocouple located close to the crack tip and a "Eurotherm" control unit capable of maintaining the desired test temperature to an accuracy of ± 1° C.

3.2. Yield stress tests

Yield stress tests were performed with dumb-bell specimens at a cross-head speed of 0.5 cm min^{-1}. The load-time plots were recorded and the maximum load was used to calculate the yield stress based on the original cross-sectional area. The dumb-bell specimens were of 11 mm thickness (for PP, 20 mm) and the other dimensions are as shown in Fig. 1c.

4. Results and discussion

A systematic series of tests were performed to establish the test method and the most suitable conditions for making valid measurements of K_{c1}. The test temperature for each polymer was selected so that a brittle failure occurred. Size effects on PMMA, PA and PVC were investigated at 20° C whereas test temperatures of − 40° C and − 60° C were chosen for testing nylon and PP, respectively. Yield stress tests were performed at the same temperatures (Table I). The critical stress intensity factor, K_c, for any particular specimen size and loading mode was determined using Equation 2 in which the gross stress, σ, was determined at the maximum load on load–time plots and the crack length, a, was taken as the initial notch length (in the case of PMMA, a slow crack growth region was easily discernible on the fracture surfaces, and the crack length was taken

TABLE I Yield stress data

Material	T (°C)	σ_y (MN m^{-2})
PMMA	+ 20	81
PVC	+ 20	58
PA	+ 20	68
Nylon	− 40	111
PP	− 60	70

as the length of the initial razor notch plus the slow crack growth [9]). The geometrical correction factor, y used for SEN specimens in tension and in bending for various span to specimen width ratios (S/W) is given in the Appendix.

4.1. Thickness effects

4.1.1. SEN three-point bend specimens

Thickness effects were examined on PMMA, PVC, PP and PA. Test specimens for each material were machined from the thickest sheet so that the sheet thickness became the width of the specimen, W. For each thickness a minimum of four specimens were prepared and notched to various a/W ratios and tested at a cross-head speed of 0.5 cm min^{-1}. Tests were performed with a span to specimen width ratio of 8:1.

The load–time plots for each specimen thickness was recorded and exhibited changes in shape as the specimen thickness was varied. In particular, the degree of non-linearity prior to the attainment of the maximum load increased as the thickness was decreased and the peaks became sharper for thicker specimens (all the specimens fractured in an unstable manner at maximum load). Examination of the fracture surfaces of the broken specimens of PA, PVC and PP showed small amounts of stress whitening at the crack tip before unstable fracture. A single crack path was observed within the whitened region but the crack bifurcated and propagated in many directions when proceeding through the rest of the specimen. Such a phenomenon is evidence of excessive strain energy being dissipated during unstable fracture.

Plots of $\sigma^2 y^2$ against a^{-1} for a given specimen thickness indicated that the instability conditions in PA, PP and PVC are well described by LEFM, in that a single parameter, K_c, independent of the crack length, described the unstable initiation condition. In the case of PMMA, the instability condition was achieved only when the crack length to specimen width ratio (a/W) was less than 0.2 (see [17]). For $a/W > 0.2$, the crack initiated and grew in a stable manner, albeit at a decreasing load.

Fig. 2 shows the effect of specimen thickness on K_c at various crack length to specimen width ratios for PMMA, PA, PP and PVC. The general trend is evident in the magnified K_c value at small thicknesses and approximately constant K_c values (K_{c1}) at large thicknesses. Observation of the load–time plots and the deformation of the specimens indicated increased non-linearity and greater lateral contraction for small thicknesses, signifying the loss of constraint and the tendency to plane-stress conditions in thinner specimens, thus tending to increase K_c. The ASTM minimum thickness requirement $B = 2.5\ (K_{c1}/\sigma_y)^2$ is shown in Fig. 2 and it appears that this requirement is a sufficient condition to ensure plane-strain conditions for valid K_{c1} testing in three-point bend specimens as it is for metals. The only exception is PP where thicknesses of about half that of the criterion still appear to give K_{c1} values.

4.1.2. SEN tension specimens

Single-edge notch tension specimens of PP and PA were cut to dimensions of 150 mm × 20 mm and notched to various a/W ratios. The specimen thickness was varied over the range 3 to 20 mm and again the sheet thickness (20 mm) was the width of the specimen. Test specimens were fractured at a constant cross-head speed of 0.5 cm min^{-1}. The fracture toughness, K_c, for each specimen was calculated using plots of $\sigma^2 y^2$ against a^{-1} and showed that a single parameter K_c described the instability condition.

The thickness dependence of K_c in tension is shown in Fig. 3, and demonstrates that the ASTM minimum thickness requirement does not provide a sufficient condition for valid K_{c1} evaluation in tension, since K_c continues to change for $B > (K_{c1}/\sigma_y)^2$. A comparison of Figs. 2 and 3 indicates that the plane-strain fracture condition in the bending mode can be achieved more readily at smaller B. This is due to the additional constraint at the crack tip in bend specimens which results in lower toughness values as compared to the tension specimen of the same thickness. The condition of the zero strain at the neutral axis in bend specimens suppresses plastic flow, thus increasing constraint.

Several models have been proposed to predict the gradual transition from the full plane-strain to full plane-stress condition. Although there is a

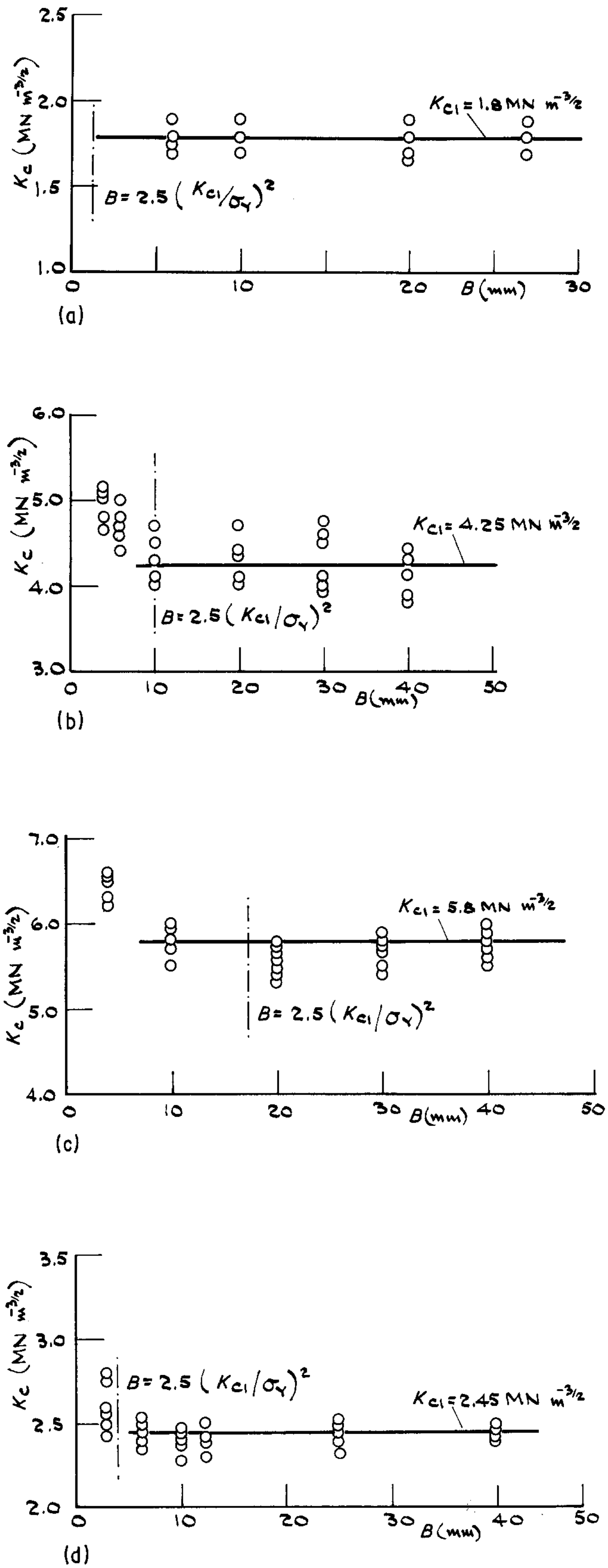


Figure 2 Effect of specimen thickness on fracture toughness in three-point bend test; (a) PMMA, $T = +20°\,C$, $\sigma_y = 81\,MN\,m^{-2}$, $W = 11\,mm$; (b) PA, $T = +20°\,C$, $\sigma_y = 68\,MN\,m^{-2}$, $W = 20\,mm$; (d) PVC, $T = +20°\,C$, $\sigma_y = 59\,MN\,m^{-2}$, $W = 12.5\,mm$.

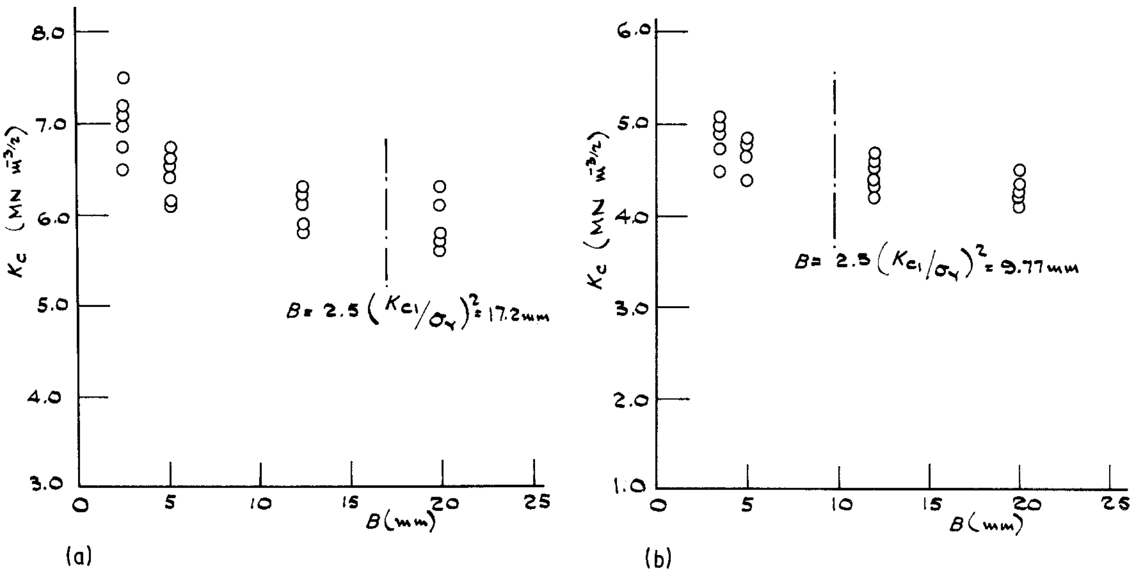


Figure 3 Fracture toughness as a function of thickness for; (a) PP at $-60°$ C; (b) PA in tension at 20° C ($W = 20$ mm).

fair qualitative understanding of the thickness effect a generally accepted quantitative model does not yet exist. However, the bimodal fracture analysis of Parvin and Williams [10], has been successful in explaining the thickness effects in tension specimens of polycarbonate and was also successful in predicting the thickness dependence of K_c for PP in tension [7]. The model assumes that a material has two fracture toughness values:

1. K_{c1}, for a plane-strain system which corresponds to the smallest amount of plastic deformation occurring under full constraint; and

2. K_{c2}, for a plane-stress system in which there is freedom from lateral constraint and consequently much greater plastic deformation.

For single-edge notch specimens, the region close to the specimen surface is under plane-stress conditions and the depth of these regions may be estimated by the plane-stress plastic zone size $r_{p2} = 1/2\pi(K_{c2}/\sigma_y)^2$. The model assumes that the measured fracture toughness K_c, in tension is a simple average of K_{c1} and K_{c2} given by;

$$BK_c = (B - 2r_{p2})K_{c1} + 2r_{p2}K_{c2}$$

i.e.

$$K_c = K_{c1} + \frac{1}{B}\left[\frac{K_{c2}^2(K_{c2} - K_{c1})}{\pi\sigma_y^2}\right]. \quad (4)$$

Provided that there is no pronounced thickness dependence of σ_y, the above relationship predicts a linear variation of K_c with inverse thickness, tending to the plane-strain value of K_{c1} as B tends to infinity. As shown in Fig. 4, the experimental data of PP and PA give such linear relationships. It is interesting to note that the extrapolated value agrees well with the K_{c1} value obtained for these polymers in three-point bend (Figs. 2b and c).

4.2. Width effects

4.2.1. SEN three-point bend specimens

Having etablished that the ASTM thickness requirement is a sufficient condition to ensure plane-strain conditions for valid K_{c1} testing in the bending mode the effect of specimen width, W, on the fracture toughness, was investigated. Test specimens of thickness 20 mm for PP and PA and 11 mm for PMMA and nylon, and widths ranging from 4 to 40 mm were machined and notched to various a/W ratios. A minimum of five specimens for each width was made and the thickness of the specimen was taken as the thickness of the sheet. Since strain rate in a three-point bend specimen is a function of the specimen width, it is preferable to ensure that a nominally constant strain rate is maintained for each width. This was done approximately using the following equation;

$$\dot{e} = 6\dot{x}(W/S^2) \quad (5)$$

where $\dot{e}$ is the strain rate at the outer fibre of an unnotched specimen, $\dot{x}$ the cross-head speed, W the specimen width and S the span. The cross-head speed and the span to width ratio (S/W) were then selected to ensure a nominally constant $\dot{e}$. The

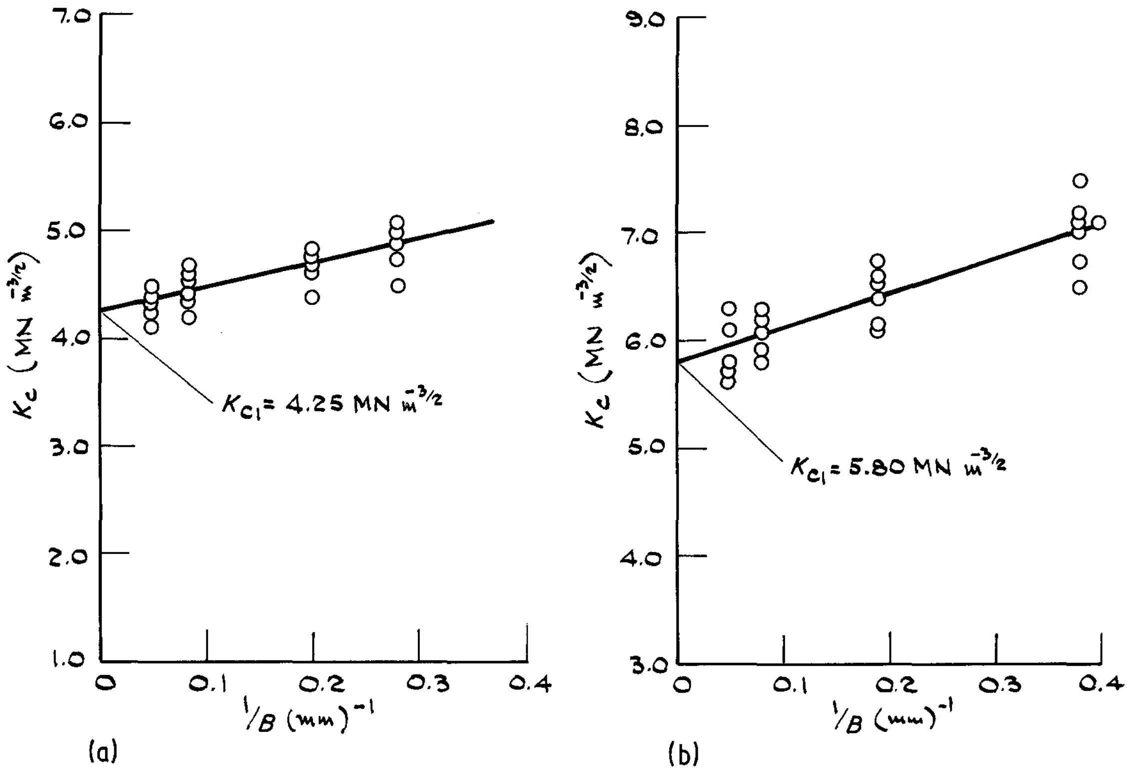


Figure 4 Fracture toughness as a function of thickness for; (a) PA at + 20° C; (b) PP at − 60° C in tension.

span was adjusted to give S/W ratios of 8:1 for widths smaller than 12 mm, and a ratio of 4:1 for widths greater than 12 mm. The cross-head speed was then selected accordingly to give $\dot{e}$ of approximately 6×10^{-4} sec^{-1} (the available cross-head speeds gave deviations of no greater than ± 50%). Load–time plots for each specimen were recorded and showed an increase in the degree of non-linearity prior to the attainment of the maximum load as the specimen width was decreased.

Fig. 5, shows that K_c is reasonably independent of crack length at each specimen width. However, there is a pronounced width dependence at low widths with K_c decreasing as W decreases. This results from the insufficient specimen width giving rise to gross yielding prior to fracture and is responsible for the non-linearity of the load–time plots and makes the LEFM increasingly invalid as W decreases indicating failure of the specimen in the post-yield regime.

The width dependence of K_c is predicted quite well by a simple yield theory of Bilby, Cottrell and Swinden (BCS) [11], as demonstrated by Chan and Williams [6]. A semi-empirical solution [12, 13] to the BCS model of a crack in a finite body subjected to an arbitrary loading condition is;

$$\sigma_c = \frac{2}{\pi}\sigma_{pc}\cos^{-1}\left[\exp\left(\frac{-\pi^2}{8}\frac{K_{c1}^2}{y^2\sigma_{pc}^2}\frac{1}{a}\right)\right], \quad (6)$$

where σ_c is the gross stress at failure, σ_{pc} the gross stress at the plastic collapse of the notched specimen, K_{c1} is the plane strain fracture toughness, y is the geometrical correction factor and a is the initial crack length.

For SEN three-point bend specimen, σ_{pc} was determined [6] from the Green and Hundy expression [14];

$$\sigma_{pc} = \begin{cases} 2.184\,\sigma_y(1-a/W)^2 & \text{plane strain} \\ 1.891\,\sigma_y(1-a/W)^2 & \text{plane stress,} \end{cases} \quad (7)$$

where σ_y is the uniaxial yield stress, a the initial crack length and W the specimen width. Equation 6, suggested that K_{c1} may be determined from tests in the post-yield regime. Indeed we may define an apparent fracture toughness, K_c, in the post-yield region and relate it to the plane strain fracture

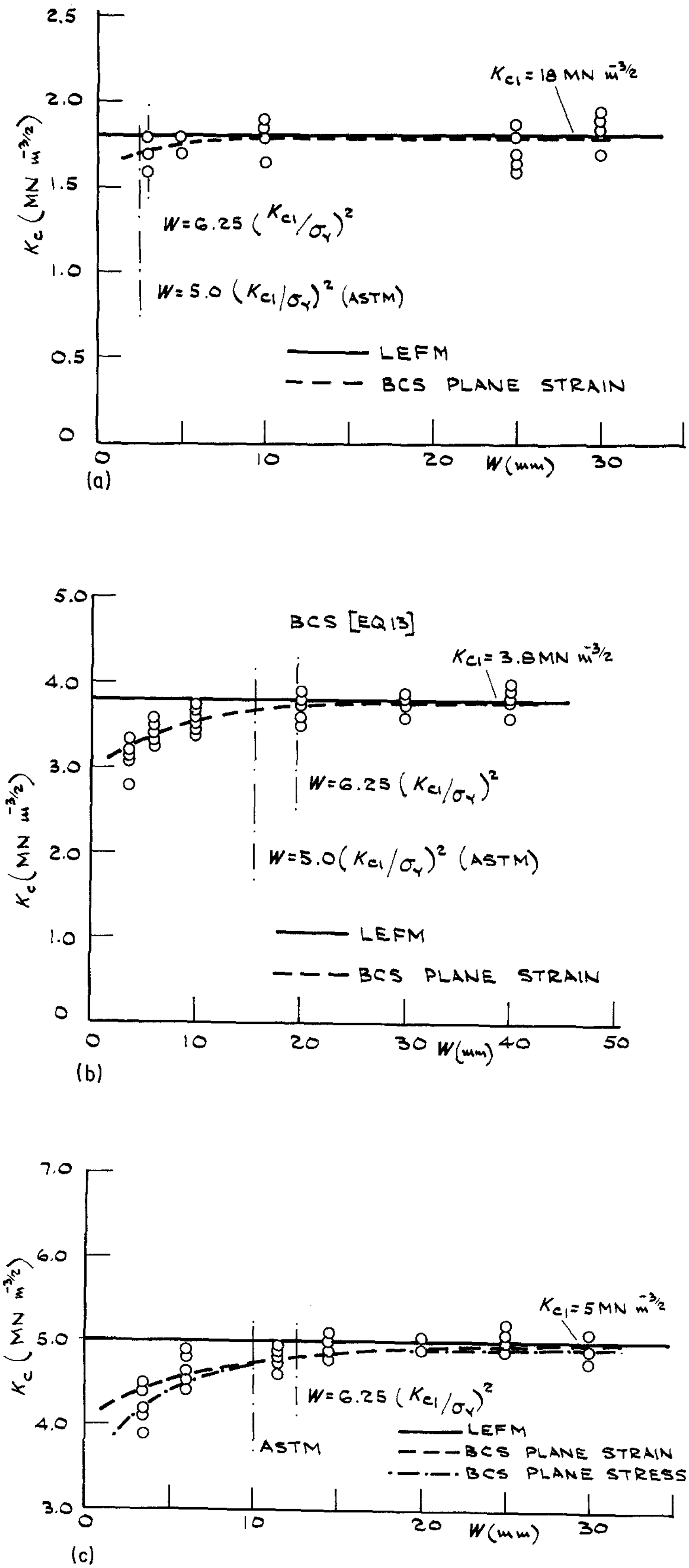
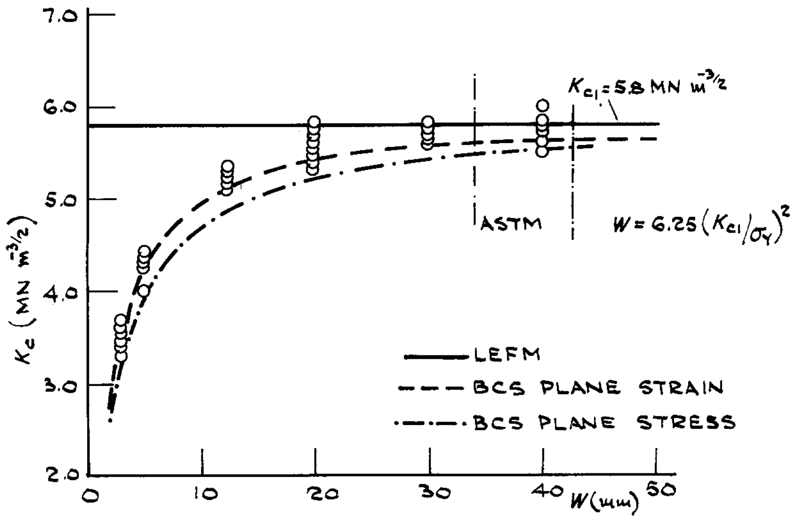


Figure 5 Effect of specimen width on K_c in three-point bend test for $W = 20$ mm. (a) PMMA, $\sigma_y = 81\,\mathrm{MN\,m^{-2}}$, $T = +20°$ C, $B = 6$ mm; (b) PA, $\sigma_y = 68\,\mathrm{MN\,m^{-2}}$, $T = +20°$ C, $B = 20$ mm; (c) Nylon, $\sigma_y = 111\,\mathrm{MN\,m^{-2}}$, $T = -40°$ C, $B = 11$ mm; (d) PP, $\sigma_y = 70\,\mathrm{MN\,m^{-2}}$, $T = -60°$ C, $B = 20$ mm.



(d)

Figure 5 Continued.

toughness through Equation 2, thus;

$$K_c = \frac{2}{\pi} y \sigma_{pc} a^{\frac{1}{2}} \cos^{-1}\left[\exp\left(\frac{-\pi^2 K_{c1}^2}{8y^2 a \sigma_{pc}^2}\right)\right]. \quad (8)$$

Equation 8 was fitted to SEN bend data to predict K_c–W behaviour in these polymers. This was done by selecting a K_{c1} value to give the best fit to the data. Results for $a/W = 0.5$ are shown in Fig. 5 and it can be seen that the experimental results agree with the BCS plane-strain solution. It must be pointed out that:

1. the predicted curves do not vary significantly for different a/W ratios; and that

2. for narrower specimens the plane-stress solution of the BCS model may be a better representation in some cases (see Figs. 5c and d). It is of interest to note that apart from PA, the K_{c1} values selected for the BCS model are in good agreement with the values obtained experimentally when thickness effects were examined (Figs. 2a to d). The discrepancy for PA, was found to be an orientational effect. A narrow white band consisting of voids was observed at the mid-thickness of the sheet. This meant that when the specimen width was taken as the thickness of the sheet, as in the case for specimen thickness studies, the voids were virtually at the neutral axis of the beam and their presence did not influence the fracture resistance of the material. However, when the specimen thickness was taken as the thickness of the sheet, the voids are aligned in the width direction at the mid-thickness. In this orientation they would be expected to effect the resistance of the material to crack propagation, and as the results show, decrease the K_{c1} value.

Having established that K_c–W can be predicted by the BCS model, it is of interest to examine the conditions for minimum specimen width W_{min}, above which LEFM could be applied to obtain a valid K_{c1} value. According to ASTM, $W_{min} = 5(K_{c1}/\sigma_y)^2$ and this condition is shown in Fig. 5.

An alternative indication of the degree of yielding at the crack tip is the net section stress, σ_N, at failure, compared to the yield stress, σ_y. In bending, σ_N can be taken as the nominal stress at the crack tip, neglecting any stress concentration effects;

$$\sigma_N = \frac{3PS}{2B(W-a)^2} \quad (9)$$

where P is fracture load.

To restrict gross yielding at the crack tip a σ_N/σ_y ratio of 0.8 has been suggested as a necessary condition [15]. Equation 9 can be expressed in terms of the specimen dimensions and $(K_{c1}/\sigma_y)^2$ by using the Brown and Srawley relation (Equation 2);

$$K_{c1} = \sigma_c y a^{\frac{1}{2}} \quad \text{and} \quad \sigma_c = \frac{3PS}{2BW^2}. \quad (10)$$

Hence:

$$\frac{K_{c1}^2}{\sigma_y^2 W} = y^2 a/W(1-a/W)^4(\sigma_N/\sigma_y)^2,$$

and applying the condition $\sigma_N/\sigma_y = 0.8$, we obtain

$$\frac{K_{c1}^2}{\sigma_y^2 W} = 0.64 y^2 a/W(1-a/W)^4. \quad (11)$$

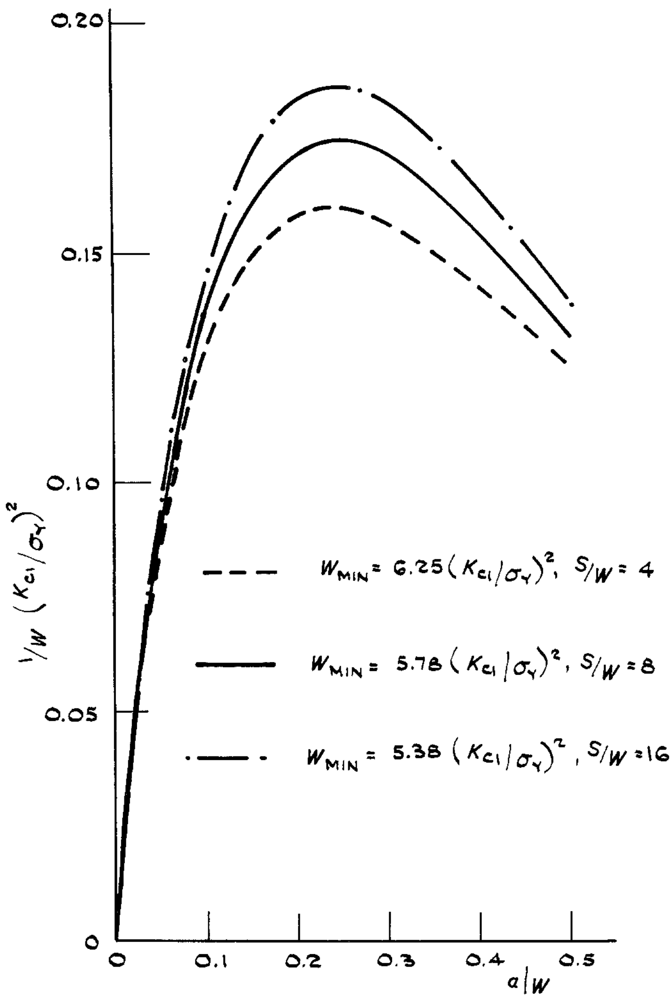


Figure 6 Yielding condition for SEN bend specimens for different span to width ratio (S/W).

This expression depends on the span-to-width ratio and is shown in Fig. 6, for $S/W = 4$, 8 and 16. It can be seen that the worst case is when $S/W = 4$, for which the minimum width, $W_{\min}$, to restrict gross yielding at the crack tip is given by, $K_{c1}^2/(\sigma_y^2 W_{\min}) = 0.16$, i.e.

$$W_{\min} = 6.25\left(\frac{K_{c1}}{\sigma_y}\right)^2. \qquad (12)$$

This condition is shown in Figs. 5a to d and seems a sufficient condition for valid K_{c1} testing of polymers in three-point-bend and is a slightly more stringent limit on $W_{\min}$ than the ASTM condition but the two are not substantially different. Again there is some evidence that PP deviates somewhat from the general pattern (Fig. 5d) in that a rather higher value of K_c is attained down to widths of half the critical value.

Another approach to estimating the minimum width requirement is to make use of the BCS model. For Equations 7 and 8, for plane-strain conditions, we arrive at the following relationship:

$$\cos\left[\frac{K_c}{\sigma_y}\frac{\alpha}{W^{\frac{1}{2}}}\right] = \exp\left[-\left(\frac{K_{c1}}{\sigma_y}\right)^2\frac{\alpha^2}{2W}\right], \qquad (13)$$

where $\alpha = \pi/2[2.184y(1-a/W)^2(a/W)^{\frac{1}{2}}]^{-1}$. It is apparent from this equation that the ratio of K_c/K_{c1} is the determining factor for estimating $W_{\min}$ for a given a/W ratio. If it is assumed that the decrease of 3% in K_c defines the necessary condition for valid K_{c1} testing then for $a/W = 0.5$, i.e. $\alpha = 1.61$, $W_{\min}$ iterated from Equation 13 gives virtually the same value as Equation 12 for all the materials tested.

4.2.2. SEN tension specimens

The effect of specimen width on K_c in tension was investigated on PP, PA and nylon. Test specimens of PA and nylon were machined from the sheet of the nominal thickness of 11 mm and for PP from

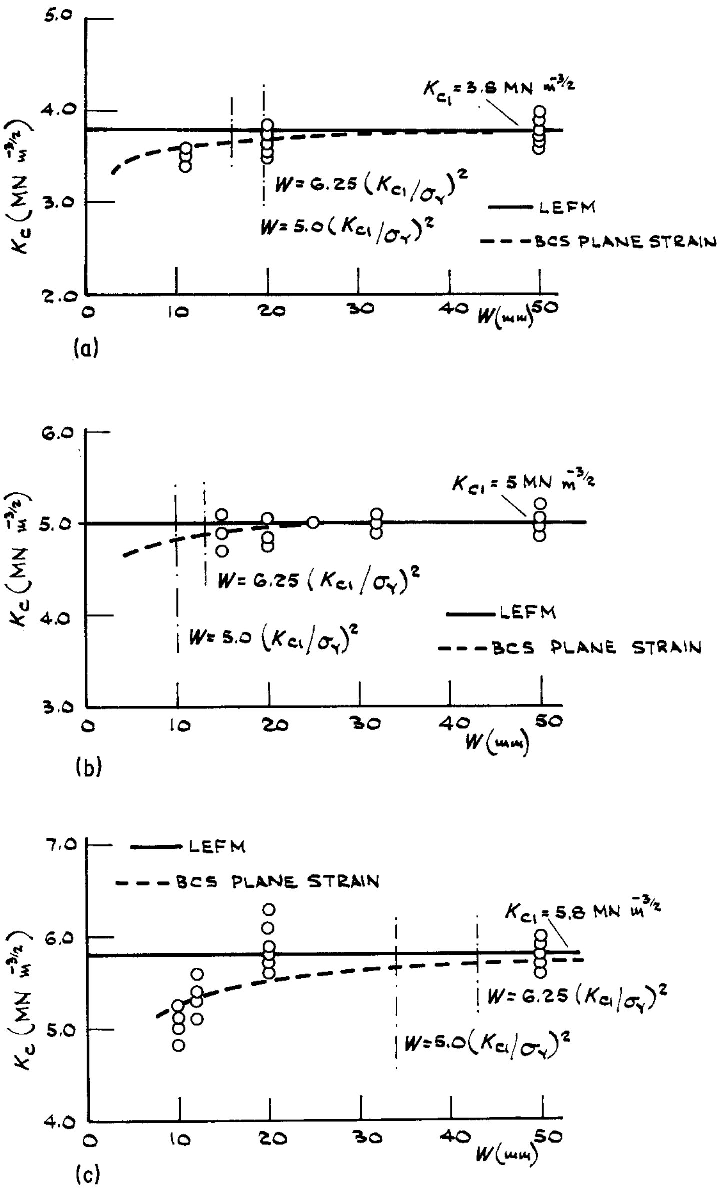


Figure 7 Fracture toughness as a function of specimen width in tension for (a) PA, $\sigma_Y = 68\,\text{MN m}^{-2}$, $B = 11$ mm; (b) Nylon, $\sigma_Y = 111\,\text{MN m}^{-2}$, $B = 11$ mm; (c) PP, $\sigma_Y = 70\,\text{MN m}^{-2}$, $B = 20$ mm.

the sheet with the nominal thickness of 20 mm and the specimen thickness was the thickness of the sheet. Specimens were then notched to various a/W ratios and tested on the Instron machine at a constant cross-head speed of 0.5 cm min^{-1}. Load–time plots for each specimen were recorded and they exhibited non-linearity prior to the attainment of the maximum load for small widths. The non-linearity became less pronounced as the specimen width was increased.

Fracture toughness values calculated using the maximum load on load–time plots were independent of the crack length. The attainment of the maximum load was always accompanied by a load report indicating the achievement of the instability condition at the maximum load. Fig. 7 shows the effect of specimen width on K_c and evidently for sufficiently wide specimens, K_c is independent of width. The ASTM, minimum width requirement (Equation 3) is also shown.

As in the case for bending, the K_c–W plot can be fairly well predicted by the BCS plane strain condition as shown in Figs. 7a to c. The plastic collapse condition can be approximated by [16];

$$\sigma_{pc} = \sigma_Y[1 - 0.809(a/W) - 1.037(a/W)^2] \qquad a/W < 0.545 \qquad (14)$$

and the minimum specimen width, above which

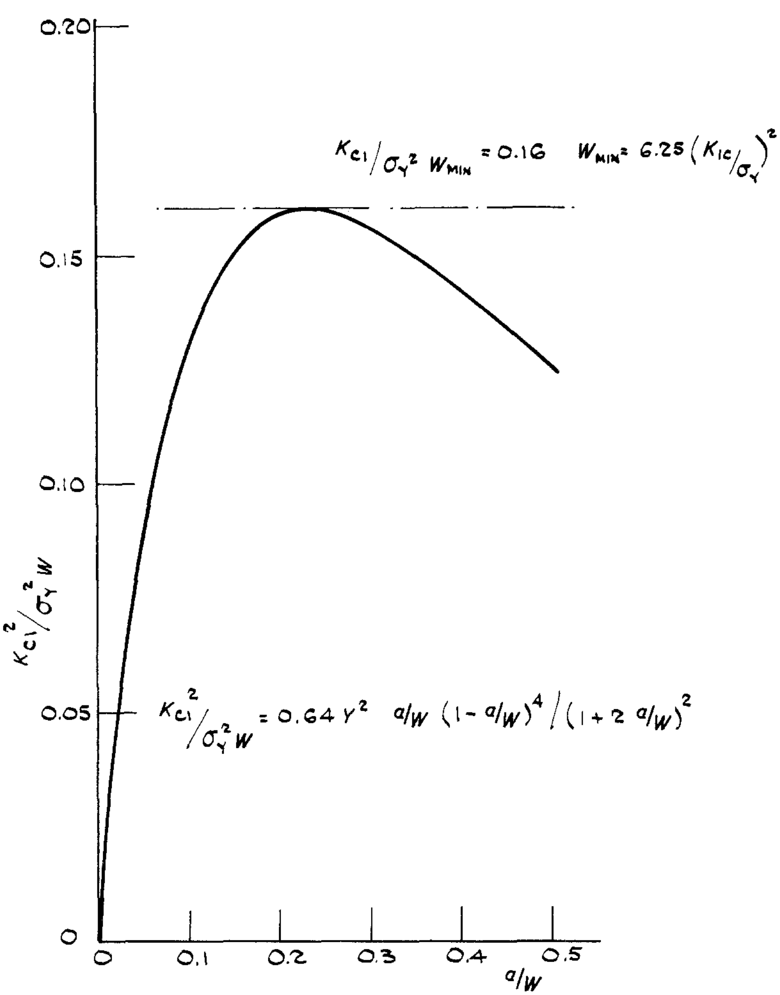


Figure 8. Yielding condition for SEN tension specimens.

LEFM can be applied to obtain a valid K_{c1} value in tension, can be estimated using the condition $\sigma_N/\sigma_y < 0.8$, The net section stress, σ_N, in tension can be defined as [17];

$$\sigma_N = \sigma_c \frac{(1-a/W)^2}{1+2a/W}, \tag{15}$$

where σ_c is the gross stress, a the crack length and W the specimen width.

Equation 15 can be expressed in terms of the specimen dimensions and the ratio $(K_{c1}/\sigma_y)^2$ using Equation 2 i.e.

$$\frac{K_{c1}^2}{\sigma_y^2 W} = 0.64 y^2 a/W(1-a/W)^4/(1+2a/W)^2. \tag{16}$$

This expression is shown in Fig. 8 and it can be seen that to restrict gross yielding at the crack tip the minimum specimen width, W_{min} is given by;

$$W_{min} = 6.25\left(\frac{K_{c1}}{\sigma_y}\right)^2. \tag{17}$$

It is interesting to note, that Equations 12 and 17 are identical, implying that although much thicker specimens are needed in tension tests to obtain a valid K_{c1} value identical widths can be used in tension and in bending to obtain plane-strain conditions. This probably reflects the fact that bending is the dominant effect in both cases. Furthermore if it is assumed that the maximum decrease of 2% in K_c defines the necessary condition for valid K_{c1} testing in tension, then the W_{min} iterated from Equation 13 for $a/W = 0.5$, gives virtually the same value as Equation 17 for all the materials tested ($\alpha = 1.30$ for $a/W = 0.5$). Again it should be noted that PP (Fig. 7c) does give satisfactory values of K_{c1} at widths of about one half of the critical value.

5. Conclusions

The results presented in this paper show that the concepts of LEFM can be applied successfully to polymers using the SEN three-point bend and SEN tension specimens to determine a valid plane-strain fracture toughness value (K_{c1}), providing certain specimen size requirements are satisfied. The present study on several polymers with varying toughnesses showed that the following models can be applied to both SEN tension and SEN bend specimens to predict variations of the toughness with the specimen dimensions, and also provide a set of requirements for determining a valid K_{c1}:

1. thickness effects. The ASTM minimum thickness requirement

$$B_{\min} = 2.5\left(\frac{K_{c1}}{\sigma_y}\right)^2$$

is a sufficient condition for determining K_{c1}, in three-point bend specimens. The condition, however, is not sufficient for valid K_{c1} testing in tension. Nevertheless, K_{c1} can be determined from SEN tension specimens using the extrapolation method of the Bimodal model (Equation 4);

2. width effects. The variation of the fracture toughness with specimen width in tension and in bending can be predicted using the BCS post-yield fracture model. Specimen width requirements for valid K_{c1} testing, based on the assumption that

$$\frac{\sigma_N}{\sigma_y} < 0.8$$

i.e.

$$W_{\min} = 6.25\left(\frac{K_{1c}}{\sigma_y}\right)^2$$

was a sufficient condition for valid K_{c1} testing in both the tension and bending modes. This condition is more severe than the condition

$$W_{\min} > 5\left(\frac{K_{c1}}{\sigma_y}\right)^2$$

suggested by ASTM and gives a better estimation of $W_{\min}$. To a first approximation, however, the ASTM criterion is satisfactory for both bending and tension tests.

In all these conditions, however, there is evidence to suggest that PP can give satisfactory values of thicknesses and widths down to half the minimum values. This may well be a consequence of crazing involved in the yielding of this material which is not at constant volume.

Appendix: K calibrations for various specimen geometries and loading conditions

1. Single-edge notched tension specimens [4]

$$K = y\frac{P}{BW}a^{\frac{1}{2}}$$

$$y = 1.99 - 0.41(a/W) + 18.7(a/W)^2 - 38.48(a/W)^3 + 53.85(a/W)^4$$

2. Single-edge notched bend specimens [4]

$$K = y\frac{6M}{BW^2}a^{\frac{1}{2}}$$

M = bending moment, where

$$y = 1.99 - 2.47(a/W) + 12.97(a/W)^2 - 23.17(a/W)^3 + 24.8(a/W)^4$$

for $S/W = 16$;

$$y = 1.96 - 2.75(a/W) + 13.66(a/W)^2 - 23.98(a/W)^3 + 25.22(a/W)^4$$

for $S/W = 8$; and

$$y = 1.93 - 3.07(a/W) + 14.53(a/W)^2 - 25.11(a/W)^3 + 25.8(a/W)^4$$

for $S/W = 4$.

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